CHIRAL INVARIANCE OF MASSIVE FERMIONS

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Abstract

We show that a massive fermion theory, while not invariant under the conventional chiral transformation, is invariant under a m-deformed chiral transformation. These transformations and the associated conserved charges are nonlocal but reduce to the usual transformations and charges when $m = 0$. The m-deformed charges commute with helicity and satisfy the conventional chiral algebra.

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I. Introduction:

Chiral invariance has played a fundamental role in the development of weak interactions [1]. As we know, the Lagrangian for a massless fermion is invariant under chiral transformations and since neutrino is believed to be massless, the requirement of chiral invariance for the two component neutrino fields leads to a V-A structure of the weak interactions [2]. Chiral symmetry has also played a prominent role in understanding the low energy properties of strong interactions through current algebra [3] and PCAC [4]. Chiral invariance of massive fermions has also been studied in the literature [5] where it has been noted that the Lagrangian for a massive fermion is invariant under the simultaneous transformations

$$\psi \rightarrow \gamma_5 \psi$$
$$m \rightarrow -m$$

(1.1)

This transformation, also known as the "mass reversal" transformation, is not widely accepted and it is believed that chiral invariance is a property of massless fermions.

It is known [6] that an interacting fermion theory at high temperature develops a temperature dependent fermion mass where the mass grows with temperature. Thus, it would appear [7] that a massless, chiral invariant theory would have its chiral symmetry broken by the temperature dependent mass. The understanding of this question becomes particularly relevant in connection with the study of chiral symmetry restoration at high temperatures where one conventionally believes that the dynamically broken chiral symmetry in QCD is restored beyond a critical temperature. As a first attempt to understand this apparent conflict, we study the chiral invariance for massive fermions at the level of the Lagrangian and the Hamiltonian.

Chiral invariance is a peculiar internal symmetry which does not allow a fermion mass term in the Lagrangian. The algebra of chiral symmetry, on the other hand, imposes no restriction on mass. In fact, being an internal symmetry, Q_5 – the generator of chiral transformation – commutes with the energy-momentum P_μ.

$$[Q_5, P_\mu] = 0 \tag{1.2}$$

Therefore,

$$[Q_5, P^2] = 0 \tag{1.3}$$

and chiral invariance would appear to impose no condition on the value of mass (which is given by the eigenvalue of the Casimir $P^2 = P_\mu P^\mu$). In fact, in section II, we will show that a massive fermion theory is invariant under m-deformed chiral transformations. These are nonlocal transformations which in the limit $m = 0$ reduce to the conventional chiral transformations. In section III, we show that the conserved charges associated with these m-deformed transformations satisfy the usual chiral algebra. We present our conclusions in section IV.

II. <u>Chiral Invariance</u>:

To appreciate the generalized chiral invariance associated with massive fermions, let us start with a free, massive fermion theory described by

$$\mathcal{L} = \overline{\psi} \left(i\gamma^\mu \partial_\mu - m \right) \psi = \overline{\psi} \left(i\gamma^0 \partial_0 + i\overrightarrow{\gamma} \cdot \overrightarrow{\nabla} - m \right) \psi \tag{2.1}$$

We use the metric $\eta^{\mu\nu} = (+, -, -, -)$ and in our convention $\gamma^{0\dagger} = \gamma^0$, $\overrightarrow{\gamma}^\dagger = -\overrightarrow{\gamma}$ and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. This Lagrangian density is not invariant under a conventional chiral transformation. However, we note that in the ultra relativistic limit, the fermion mass becomes negligible and a chiral invariance would result. Keeping this in mind, we define a new field variable through a generalized Foldy-Wouthuysen transformation [8] (more accurately a Cini-Touschek transformation) as

$$\psi(x) = U(\overrightarrow{\nabla})\psi'(x) \tag{2.2}$$

where

$$\begin{aligned} U(\overrightarrow{\nabla}) = &\frac{1}{[2(m^2 - \overrightarrow{\nabla}^2)^{1/2}((m^2 - \overrightarrow{\nabla}^2)^{1/2} + (-\overrightarrow{\nabla}^2)^{1/2})]^{1/2}} \\ &\times \left((m^2 - \overrightarrow{\nabla}^2)^{1/2} + (-\overrightarrow{\nabla}^2)^{1/2} - im\,\frac{\overrightarrow{\gamma} \cdot \overrightarrow{\nabla}}{(-\overrightarrow{\nabla}^2)^{1/2}} \right) \end{aligned} \tag{2.3}$$

and the derivatives are all acting to the right. We note here that conventionally such a transformation is written in momentum space as

$$U = e^{\frac{1}{2m}\,\overrightarrow{\gamma} \cdot \overrightarrow{p}\,\theta\left(\frac{p}{m}\right)} \tag{2.4}$$

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with

$$\tan\left(\frac{p\theta}{m}\right) = \frac{m}{p} \qquad\qquad (p = |\overrightarrow{p}|) \tag{2.5}$$

Equivalently, one can also write

$$U = e^{\frac{1}{2p}\,\overrightarrow{\gamma}\cdot\overrightarrow{p}\,\theta(\frac{p}{m})} \tag{2.6}$$

with

$$\tan\theta = \frac{m}{p} \tag{2.7}$$

However, we choose to work in the coordinate space and one can check results when necessary by going over to the momentum space.

Under the field redefinition of Eq. (2.3), the Lagrangian density of Eq. (2.1) takes the form (up to surface terms)

$$\begin{aligned}
\mathcal{L} &= \overline{\psi}\big(i\gamma^0\partial_0 + i\overrightarrow{\gamma}\cdot\overrightarrow{\nabla} - m\big)\psi \\
&= \overline{\psi}'\left(i\gamma^0\partial_0 + i\overrightarrow{\gamma}\cdot\overrightarrow{\nabla}\,\frac{(m^2 - \overrightarrow{\nabla}^2)^{1/2}}{(-\overrightarrow{\nabla}^2)^{1/2}}\right)\psi' \\
&= \overline{\psi}'\big(i\gamma^0\partial_0 + i\overrightarrow{\gamma}\cdot\overrightarrow{\nabla}\hat{O}\big)\psi'
\end{aligned} \tag{2.8}$$

where we have defined

$$\hat{O} = \frac{(m^2 - \overrightarrow{\nabla}^2)^{1/2}}{(-\overrightarrow{\nabla}^2)^{1/2}} \tag{2.9}$$

We note that under the field redefinition, the Lagrangian has become nonlocal, but it is now manifestly chiral invariant. Before proceeding, we note that the equations of motion in terms of the primed and unprimed variables take the form

$$\begin{aligned}
\big(i\gamma^0\partial_0 + i\overrightarrow{\gamma}\cdot\overrightarrow{\nabla} - m\big)\psi &= 0 \\
\overline{\psi}\big(i\gamma^0\overleftarrow{\partial}_0 + i\overrightarrow{\gamma}\cdot\overleftarrow{\nabla} + m\big) &= 0
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
\big(i\gamma^0\partial_0 + i\overrightarrow{\gamma}\cdot\overrightarrow{\nabla}\hat{O}\big)\psi' &= 0 \\
\overline{\psi}'\big(i\gamma^0\overleftarrow{\partial}_0 + i\overrightarrow{\gamma}\cdot\overleftarrow{\nabla}\overleftarrow{\hat{O}}\big) &= 0
\end{aligned} \tag{2.11}$$

where $\overleftarrow{\hat{O}}$ stands for derivatives acting to the left.

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Although there is no mass term in Eq. (2.11), they really describe massive particles. This is easily seen in momentum space where the equation has the form

$$\left(\gamma^0 p^0 - \overrightarrow{\gamma} \cdot \overrightarrow{p} \, \frac{(m^2 + p^2)^{1/2}}{p}\right)\psi' = 0$$

$$\text{or,} \quad \left(\gamma^0 p^0 - \overrightarrow{\gamma} \cdot \overrightarrow{p} \, \frac{(m^2 + p^2)^{1/2}}{p}\right)^2 \psi' = 0 \tag{2.12}$$

$$\text{or,} \quad \left(p^{0^2} - (m^2 + p^2)\right)\psi' = 0$$

$$\text{or,} \quad p^0 = \pm\left(m^2 + p^2\right)^{1/2}$$

On the other hand, the Lagrangian of Eq. (2.8) is invariant under the global chiral transformations

$$\psi' \rightarrow e^{i\alpha\gamma_5}\psi'$$

$$\overline{\psi}' \rightarrow \overline{\psi}' e^{i\alpha\gamma_5} \tag{2.13}$$

To understand the symmetry further, we need to construct the charges and study their algebra. To this end, we note that because the Lagrangian in Eq. (2.8) is nonlocal, the derivation of the Noether current has to be carried out carefully. In fact, since the nontrivial operator $\hat{O}$ is not linear, namely, since

$$\hat{O}(AB) \neq (\hat{O}A)B + A(\hat{O}B) \tag{2.14}$$

the continuity equation under an infinitesimal symmetry transformation for the present case can be shown to be

$$\partial_0\left(\frac{\partial\mathcal{L}}{\partial\partial_0\psi'(x)} \, \delta_\epsilon\psi'(x)\right) + \overrightarrow{\nabla} \cdot \left(\frac{\partial\mathcal{L}}{\partial\overrightarrow{\nabla}\hat{O}\psi'(x)} \, \delta_\epsilon\psi'(x)\right)$$
$$= \left(\overrightarrow{\nabla} \cdot \frac{\partial\mathcal{L}}{\partial\overrightarrow{\nabla}\hat{O}\psi'(x)}\right) \overset{\leftrightarrow}{\hat{O}} \delta_\epsilon\psi'(x) \tag{2.15}$$

Here $\overset{\leftrightarrow}{\hat{O}} = \overset{\rightarrow}{\hat{O}} - \overset{\leftarrow}{\hat{O}}$. For the infinitesimal chiral transformations in Eq. (2.13) then, we can define

$$J_5'^0 = \overline{\psi}'(x)\gamma_5\gamma^0\psi'(x)$$

$$\overrightarrow{J}_5' = \overline{\psi}'(x)\gamma_5\overrightarrow{\gamma}\hat{O}\psi'(x) \tag{2.16}$$

$$J_5' = \overline{\psi}'(x)\gamma_5\overrightarrow{\gamma} \cdot \overset{\leftrightarrow}{\nabla} \overset{\leftrightarrow}{\hat{O}} \psi'(x)$$

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That these operators satisfy the continuity equation (see Eq. (2.15))

$$\partial_0 J_5'^0 + \vec{\nabla} \cdot \vec{J}_5' = J_5'$$

(2.17)

can be easily checked using the equations of motion in Eq. (2.11). The noncovariant nature of the currents is understandable since the field redefinition in Eq. (2.2) or (2.4) or (2.6) is manifestly noncovariant. However, the inhomogeneous term in Eq. (2.17) makes one wonder whether the charge will be time independent. We note from the structure of $J_5'(x)$ that

$$\int d^3x \ J_5'(x) = 0$$

(2.18)

(This can be seen more easily in momentum space if the coordinate space derivation appears formal.) Therefore, $J_5'(x)$ can really be expressed as a total divergence although the expression is not simple. Therefore, we continue to use the form of the continuity equation in (2.17) and simply note that because of Eq. (2.18), the charge

$$Q_5' = \int d^3x \ \overline{\psi}'(x)\gamma_5\gamma^0\psi'(x)$$

(2.19)

is a constant of motion.

The invariance of the action in terms of the original variables can now be easily constructed. Under

$$\begin{aligned}
\psi(x) \to & U(\vec{\nabla})e^{i\alpha\gamma_5}U(-\vec{\nabla})\psi(x) \\
= & \left(\cos\alpha + i\gamma_5 \sin\alpha \ \frac{(-\vec{\nabla}^2)^{1/2}}{(m^2 - \vec{\nabla}^2)^{1/2}} \left(1 - im \ \frac{\vec{\gamma}\cdot\vec{\nabla}}{\vec{\nabla}^2} \right) \right)\psi(x)
\end{aligned}$$

(2.20)

it can be easily checked that the action associated with Eq. (2.1) is invariant. This is a manifestly nonlocal generalization of the conventional chiral transformation and we note that when $m = 0$ this reduces to the usual chiral transformation associated with a massless field. We call these the m-deformed chiral transformations and we note that the massive fermion theory of Eq. (2.1) is invariant under such transformations.

The conserved currents associated with these transformations can now be derived. However, because the transformations are nonlocal, the derivation has to be carried out

with care. Alternately, we can obtain the currents from Eq. (2.17) through the inverse field redefinition. In fact, it is easy to check using Eq. (2.10) that

$$J_5^0 = \overline{\psi}(x)U(\overleftarrow{\nabla})\gamma_5\gamma^0 U(-\overrightarrow{\nabla})\psi(x)$$
$$\overrightarrow{J}_5 = \overline{\psi}(x)U(\overleftarrow{\nabla})\gamma_5\overrightarrow{\gamma}\hat{O}U(-\overrightarrow{\nabla})\psi(x) \tag{2.21}$$
$$J_5 = \overline{\psi}(x)U(\overleftarrow{\nabla})\gamma_5\overrightarrow{\gamma}\cdot\overrightarrow{\nabla}\overset{\leftrightarrow}{\hat{O}}U(-\overrightarrow{\nabla})\psi(x)$$

satisfy the continuity equation

$$\partial_0 J_5^0 + \overrightarrow{\nabla}\cdot\overrightarrow{J}_5 = J_5 \tag{2.22}$$

The charge

$$Q_5 = \int d^3x\, J_5^0(x)$$
$$= \int d^3x\, \overline{\psi}(x)\gamma_5\gamma^0 \frac{(-\overrightarrow{\nabla}^2)^{1/2}}{(m^2-\overrightarrow{\nabla}^2)^{1/2}}\left(1 - im\,\frac{\overrightarrow{\gamma}\cdot\overrightarrow{\nabla}}{\overrightarrow{\nabla}^2}\right)\psi(x) \tag{2.23}$$

can be seen to reduce to the usual chiral charge when $m = 0$ and, therefore, can be thought of as a m-deformed chiral charge. It can also be easily seen to be a constant of motion in a variety of ways. In fact, from Eq. (2.20) we note that the first quantized generator of symmetry can be identified with

$$Q_5 = \gamma_5\left(\frac{(-\overrightarrow{\nabla}^2)^{1/2}}{(m^2-\overrightarrow{\nabla}^2)^{1/2}} + im\,\frac{\overrightarrow{\gamma}\cdot\overrightarrow{\nabla}}{(m^2-\overrightarrow{\nabla}^2)^{1/2}(-\overrightarrow{\nabla}^2)^{1/2}}\right) \tag{2.24}$$

It is straightforward to check that this commutes with the first quantized Hamiltonian

$$H = -i\gamma^0\overrightarrow{\gamma}\cdot\overrightarrow{\nabla} + m\gamma^0 \tag{2.25}$$

$$[Q_5, H] = 0 \tag{2.26}$$

Let us also note that the helicity in the first quantized theory is defined to be

$$h = -i\gamma_5\gamma^0\frac{\overrightarrow{\gamma}\cdot\overrightarrow{\nabla}}{(-\overrightarrow{\nabla}^2)^{1/2}} \tag{2.27}$$

It follows then that

$$[Q_5, h] = 0 \tag{2.28}$$

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Namely, the m-deformed chiral charge continues to commute with helicity. However, we note that

$$[Q_5, \gamma_5] = -2im \frac{\overrightarrow{\gamma} \cdot \overrightarrow{\nabla}}{(m^2 - \overrightarrow{\nabla}^2)^{1/2}(-\overrightarrow{\nabla}^2)^{1/2}} \tag{2.29}$$

and the commutator vanishes for $m = 0$.

III. <u>Algebra of Charges</u>:

To study the algebra of the m-deformed chiral charges, let us consider, for simplicity, a free, massive fermion theory where the fermion fields belong to the fundamental representation of SU(2).

$$\mathcal{L} = \overline{\psi}\left(i\slashed{\partial} - m\right)\psi = \overline{\psi}\left(i\gamma^0 \partial_0 + i\overrightarrow{\gamma} \cdot \overrightarrow{\nabla} - m\right)\psi \tag{3.1}$$

Once again, if we redefine the field variables as in Eqs. (2.2) and (2.3), the Lagrangian will have the form

$$\mathcal{L} = \overline{\psi}'\left(i\gamma^0 \partial_0 + i\overrightarrow{\gamma} \cdot \overrightarrow{\nabla} \frac{(m^2 - \overrightarrow{\nabla}^2)^{1/2}}{(-\overrightarrow{\nabla}^2)^{1/2}}\right)\psi' \tag{3.2}$$

where $\psi'(x)$ is a doublet of SU(2). This Lagrangian is invariant under global $\text{SU}_\text{V}(2)$ and $\text{SU}_\text{A}(2)$ transformations defined by

$$\begin{aligned}
\psi'(x) &\rightarrow e^{\frac{i}{2}\,\overrightarrow{\alpha}\cdot\overrightarrow{\tau}}\,\psi'(x) \\
\psi'(x) &\rightarrow e^{\frac{i}{2}\,\gamma_5\overrightarrow{\beta}\cdot\overrightarrow{\tau}}\,\psi'(x)
\end{aligned} \tag{3.3}$$

where τ^a's represent the Pauli matrices – the generators of SU(2). Following the discussions of the previous section, we can construct the conserved charges associated with these transformations and they are given by

$$Q'^a = \int d^3x \, \overline{\psi}'(x) \gamma^0 \left(\frac{\tau^a}{2}\right) \psi'(x)$$

$$a = 1, 2, 3 \qquad (3.4)$$

$$Q_5'^a = \int d^3x \, \overline{\psi}'(x) \gamma_5 \gamma^0 \left(\frac{\tau^a}{2}\right) \psi'(x)$$

Using the canonical anticommutation relations following from Eq. (3.2), it can be easily checked that these charges satisfy the conventional chiral algebra [9]

$$\left[Q'^a, Q'^b\right] = i \, \epsilon^{abc} Q'^c$$

$$\left[Q'^a, Q_5'^b\right] = i \, \epsilon^{abc} Q_5'^c \qquad (3.5)$$

$$\left[Q_5'^a, Q_5'^b\right] = i \, \epsilon^{abc} Q'^c$$

The Lagrangian of Eq. (3.1) is, of course, invariant under global $\mathrm{SU_V}(2)$ transformations. But it is also invariant under the m-deformed $\mathrm{SU_A}(2)$ transformations

$$\psi(x) \rightarrow \left(\cos\frac{\beta}{2} + i \, \gamma_5 \hat{\beta} \cdot \overrightarrow{\tau} \sin\frac{\beta}{2} \frac{(-\overrightarrow{\nabla}^2)^{1/2}}{(m^2 - \overrightarrow{\nabla}^2)^{1/2}} \left(1 - im\frac{\overrightarrow{\gamma} \cdot \overrightarrow{\nabla}}{\overrightarrow{\nabla}^2} \right) \right) \psi(x) \qquad (3.6)$$

The conserved charges associated with these transformations can again be constructed and have the form

$$Q^a = \int d^3x \, \overline{\psi}(x) \gamma^0 \left(\frac{\tau^a}{2}\right) \psi(x)$$

$$Q_5^a = \int d^3x \, \overline{\psi}(x) \gamma_5 \gamma^0 \left(\frac{\tau^a}{2}\right) \frac{(-\overrightarrow{\nabla}^2)^{1/2}}{(m^2 - \overrightarrow{\nabla}^2)^{1/2}} \left(1 - im\frac{\overrightarrow{\gamma} \cdot \overrightarrow{\nabla}}{\overrightarrow{\nabla}^2} \right) \psi(x) \qquad (3.7)$$

It is now straightforward to check, from the fundamental anticommutation relations following from Eq. (3.1), that

$$\left[Q^a, Q^b\right] = i \, \epsilon^{abc} Q^c$$

$$\left[Q^a, Q_5^b\right] = i \, \epsilon^{abc} Q_5^c \qquad (3.8)$$

$$\left[Q_5^a, Q_5^b\right] = i \, \epsilon^{abc} Q^c$$

Namely, the algebra of the m-deformed generators continues to be the usual chiral algebra.

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Finally, let us note that the operators

$$S' = \frac{1}{2} \int d^3x \, \overline{\psi}'(x)\psi'(x)$$

$$P' = -\frac{i}{2} \int d^3x \, \overline{\psi}'(x)\gamma_5\psi'(x) \tag{3.9}$$

$$\tilde{Q}'_5 = \frac{1}{2} \, Q'_5 = \frac{1}{2} \int d^3x \, \overline{\psi}'(x)\gamma_5\gamma^0\psi'(x)$$

satisfy the equal time SU(2) algebra

$$[S', P'] = i \, \tilde{Q}'_5$$

$$[S', \tilde{Q}'_5] = -i \, P' \tag{3.10}$$

$$[P', \tilde{Q}'_5] = i \, S'$$

The last two relations of Eq. (3.10), of course, represent the changes in S' and P' under a chiral transformation and we recognize the closed, equal time algebra to correspond to a SU(2) algebra [10]. Correspondingly, in terms of the original variables, the operators

$$S = \frac{1}{2} \int d^3x \, \overline{\psi}(x) \frac{(-\overrightarrow{\nabla}^2)^{1/2}}{(m^2 - \overrightarrow{\nabla}^2)^{1/2}} \left(1 - im\frac{\overrightarrow{\gamma} \cdot \overrightarrow{\nabla}}{\overrightarrow{\nabla}^2}\right)\psi(x)$$

$$P = -\frac{i}{2} \int d^3x \, \overline{\psi}(x)\gamma_5\psi(x) \tag{3.11}$$

$$\tilde{Q}_5 = \frac{1}{2} \, Q_5 = \frac{1}{2} \int d^3x \, \overline{\psi}(x)\gamma_5\gamma^0 \frac{(-\overrightarrow{\nabla}^2)^{1/2}}{(m^2 - \overrightarrow{\nabla}^2)^{1/2}} \left(1 - im \, \frac{\overrightarrow{\gamma} \cdot \overrightarrow{\nabla}}{\overrightarrow{\nabla}^2}\right)\psi(x)$$

satisfy the equal time SU(2) algebra

$$[S, P] = i \, \tilde{Q}_5$$

$$[S, \tilde{Q}_5] = -i \, P \tag{3.12}$$

$$[P, \tilde{Q}_5] = i \, S$$

IV. Conclusion:

Although a massive fermion theory is not invariant under a conventional, global chiral transformation, we have shown that it is invariant under a m-deformed chiral transformation. (This is quite similar to the fact that a spontaneously broken massive gauge theory

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has a m-deformed gauge invariance (transformations depending on mass or vacuum expectation value) at the Lagrangian level. The difference is that the m-deformed gauge transformations are local and further, since the number of degrees of freedom is not the same for $m = 0$ and $m \neq 0$, in the case of a gauge theory, one needs additional fields (namely, the Higgs field) for this.) These transformations as well as the corresponding conserved charges are manifestly nonlocal but reduce to the usual transformations and charges when $m = 0$. The m-deformed charges continue to commute with the helicity operator and satisfy the usual chiral algebra. A massive fermion theory is also known to have a second γ_5 invariance [11] where, however, the generators have explicit time dependence [12]. Our discussion so far has been at the level of operators, the Lagrangian and the Hamiltonian. The question of chiral symmetry restoration at high temperature needs further careful study of the structure of the Hilbert space and will be reported in a later publication.

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